5.1 Work and the Transformation of Energy

SECTION EXPECTATIONS

- Define and describe concepts and units related to energy forms and conservation.
- Analyze and explain common situations using the work-energy theorem.
- Investigate the law of conservation of energy experimentally.

KEY TERMS

- energy
- work
- work-kinetic energy theorem
- work-energy theorem
- conservation of mechanical energy

In the introduction to this chapter, you read about some different types of energy transfers and transformations. You might recall from previous science courses that the two mechanisms by which energy is transferred from one system to another are work and heat. In fact, **energy** is often defined as the ability to do work. In this section, you will focus on work, extending your knowledge and your ability to make predictions about work and solve problems involving work as the transfer of mechanical energy from one system to another.

Characterizing Work

What is work? How do you know if one object or system is doing work on another? If work is being done, how much work is done? In physics, these questions are easier to answer than in everyday life. If an object or system, such as your body, exerts a force on an object and that force causes the object's position to change, you are doing **work** on the object.

The most direct way to express work mathematically is with the equation $W = F\Delta d\cos\theta$, where F is the magnitude of the force doing work on an object, Δd is the magnitude of the displacement caused by the force, and θ is the angle between the vectors for force and displacement. Notice that the force, F, and displacement, Δd, do not have vector notations. The reason for the omission of the vector symbols is that work, W, is a scalar quantity and is the scalar product of the vectors $\vec{F}$ and $\Delta\vec{d}$. Since the product of the vectors is a scalar quantity, the directions of the force and displacement do not determine a final direction of their product. To understand why $\cos\theta$ is included in the equation, study Figure 5.1.

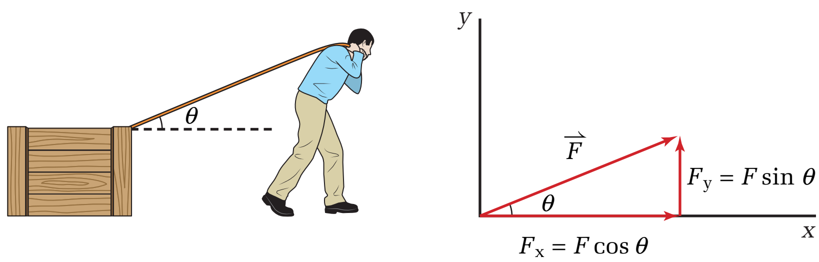
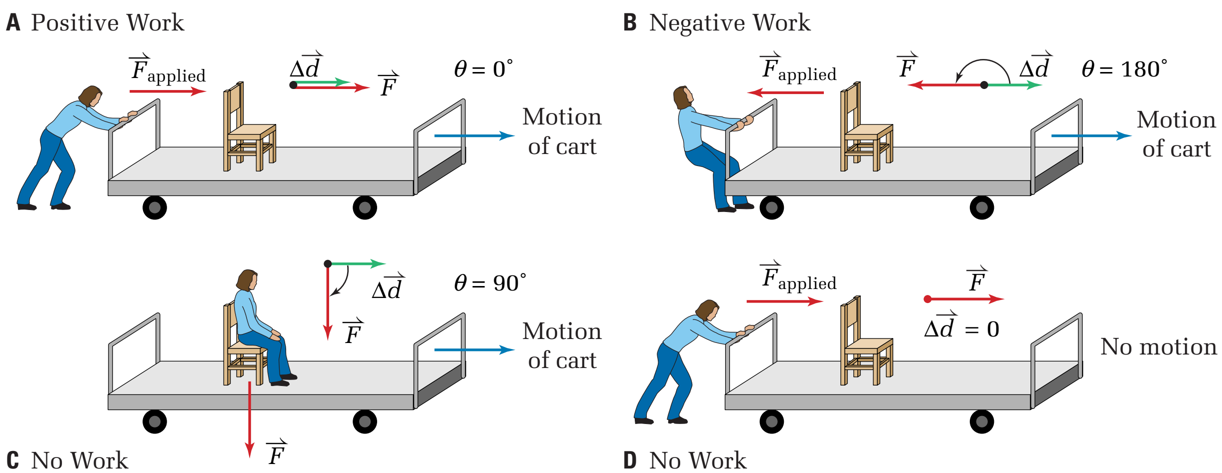


Figure 5.1 The only component of the force acting on an object that does work is the component that is parallel to the direction of the displacement.

In Figure 5.1, you see a person pulling a crate with a force that is at an angle, θ, relative to the direction of the motion. Only part of that force is actually doing work on the crate. In the diagram beside the sketch, you can see that the x-component (horizontal) of the force has a magnitude $F\cos\theta$. This component is in the direction of the motion and is the only component that is doing work. The y-component (vertical) is perpendicular to the direction of the motion and does no work on the crate.

Figure 5.2 shows four special cases that will clarify the question of whether work is being done by a force. In part (A), a person is pushing a cart with a force ($\vec{F}$) that is in the same direction as the motion of the cart. The angle between the force and the displacement is zero, so $\cos\theta = \cos 0 = 1$ and the work is $W = F\Delta d$. When the force and the displacement are in the same direction, the entire force is doing work. In this case, the cart is speeding up, so its kinetic energy is increasing. The work that the person is doing on the cart is transferring energy to the cart, so positive work is being done on the cart.

In part (B) of Figure 5.2, the cart has kinetic energy and is moving forward. The person is pulling on the cart to slow it down. Notice that the direction of the force that the person is exerting on the cart is opposite to the direction of the motion. The angle θ is 180°, so $\cos\theta = -1$ and, therefore, $W = -F\Delta d$. Just as the results indicate, the person is doing negative work on the cart by slowing it down and reducing its kinetic energy.



In part (C) of Figure 5.2, the person is sitting on the cart, exerting a downward force on it. The angle θ is 90°, so $\cos\theta = \cos 90^\circ = 0$ and the work is $W = F\Delta d(0) = 0$. Even though the cart is moving, the force that the person is exerting is not doing work, because it is not directly affecting the horizontal motion of the cart. Notice that if you use the equation for work properly, the term $\cos\theta$ will tell you whether the work is positive, negative, or zero.

MATH LINK

In mathematics, a scalar product is also called a “dot product” and the equation for work is written as $W = \vec{F} \cdot \Delta\vec{d}$ The magnitude of a dot product is always the product of the magnitudes of the two vectors and the cosine of the angle between them.

Figure 5.2 In order to do work, the force must be acting in a direction parallel to the displacement.

Finally, in part (D), the person is pushing on the cart, but the cart is stuck and will not move. Even though the person is exerting a force on the cart, the person is not doing work on the cart, because the displacement is zero.

DEFINING WORK

Work is the product of the force, the displacement, and the cosine of the angle between the force and displacement vectors.

$$W = F\Delta d \cos \theta$$

Quantity	Symbol	SI unit
work	W	J (joules)
force	F	N (newtons)
displacement	Δd	m (metres)
angle between force and displacement	θ	degrees (The cosine of an angle is a number and has no units.)

Unit Analysis

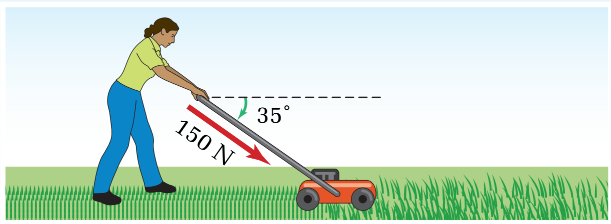
(joule) = (newton)(metre) $\text{J} = \text{N} \cdot \text{m}$

A newton · metre is equivalent to a joule.

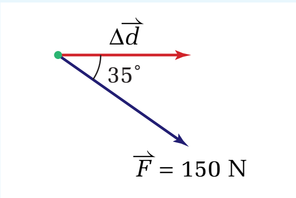
SAMPLE PROBLEM

Working on the Lawn

A woman pushes a lawnmower with a force of 150 N at an angle of 35° down from the horizontal. The lawn is 10.0 m wide and requires 15 complete trips across and back. How much work does she do?



Conceptualize the Problem



- Draw a sketch to show the relationship between the force and the motion.
- A *force* is acting at an *angle* to the *direction of motion*.
- Since a *component* of the force is in the *direction of the motion*, the force is *doing work* on the lawnmower.
- *Work* done can be determined from the general work equation.

Identify the Goal

The work, W, done by the woman on the lawnmower

Identify the Variables and Constants

Known	**Unknown**
$\vec{F} = 150\ \text{N}$	Δd
$\theta = 35^\circ$	W
width of lawn = 10.0 m	
15 trips	

Develop a Strategy

Determine the total distance over which the force acted.

$$\Delta d = 2(10.0\ \text{m})(15)$$
$$\Delta d = 300\ \text{m}$$

Use the general work equation.

$$W = F\Delta d \cos\theta$$

Substitute and solve.

$$W = (150\ \text{N})(300\ \text{m})\cos 35^\circ$$
$$W = 36\ 861.8\ \text{J}$$
$$W \cong 3.7 \times 10^4\ \text{J}$$

The work done by the woman on the lawnmower is 3.7×10^4 J.

Validate the Solution

If the force had been horizontal, then the work done would have been $300\ \text{m} \times 150\ \text{N}$, which equals 45 000 J. Because the force is at an angle to the direction of motion, the work done is less than this value.

PRACTICE PROBLEMS

1. A man pulls with a force of 100.0 N at an angle of 25° up from the horizontal on a sled that is moving horizontally. If the sled moves a distance of 200.0 m, how much work does the man do on the sled?
2. A tow truck does 42.0 MJ of work on a car while pulling it with a force of 3.50 kN exerted upward at 10.0° to the horizontal. If the car moves horizontally, how far was it towed?
3. A kite moves 14.0 m horizontally while pulled by a string. If the string did 60.0 J of work on the kite while exerting a force of 8.2 N, what angle did the string make with the vertical?

Work and Kinetic Energy

In the examples you just examined, you determined the amount of work that was done on several objects. Now, consider the form of the energy that is given to an object on which work is done and the relationship to the work that was done. First, look at a situation in which all of the work done on a cart transfers only kinetic energy to the cart. Imagine that a cart is rolling horizontally to the right with a speed of v_i when a force is exerted on it in the direction of motion, as shown in Figure 5.3. The force acts

over a displacement of $\Delta\vec{d}$. Since all of the motion is horizontal, there will be no changes in gravitational potential energy. Assume also that friction is negligible. Study Figure 5.3 and then examine the steps that follow the illustration.

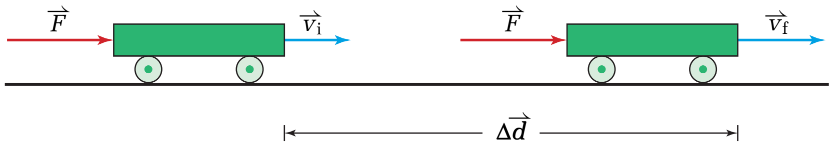


Figure 5.3 All of the work being done on the cart is increasing the cart's kinetic energy.

- Write the equation for work. From Figure 5.3, you can see that the angle between the force vector and the displacement vector is zero. Use this information to simplify the equation.

$$W = F\Delta d\cos\theta$$
$$W = F\Delta d\cos 0^\circ$$
$$\cos 0^\circ = 1$$
$$W = F\Delta d$$

- Recall the expression for force from Newton's second law. Substitute the expression for F into the equation for work. Since work is a scalar quantity, do not use vector symbols.

$$F = ma$$
$$W = ma\Delta d$$

- Write the kinematic equation that relates initial velocity, final velocity, displacement, and acceleration. Solve that equation for displacement.

$$v_{\mathrm{f}}^2 = v_{\mathrm{i}}^2 + 2a\Delta d$$
$$\Delta d = \frac{v_{\mathrm{f}}^2 - v_{\mathrm{i}}^2}{2a}$$

- Substitute the expression for displacement into the equation for work. Simplify the expression.

$$W = ma\left(\frac{v_{\mathrm{f}}^2 - v_{\mathrm{i}}^2}{2a}\right)$$
$$W = \frac{m(v_{\mathrm{f}}^2 - v_{\mathrm{i}}^2)}{2}$$

- Expand the equation.

$$W = \frac{1}{2}mv_{\mathrm{f}}^2 - \frac{1}{2}mv_{\mathrm{i}}^2$$

- Recognize the expression $\frac{1}{2}mv^2$ as kinetic energy.

$$W = E_{\mathrm{kf}} - E_{\mathrm{ki}}$$
$$W = \Delta E_{\mathrm{k}}$$

The work done on the cart is equal to the change in the kinetic energy of the cart. You can now generalize and state that, when work is done on an object in which the force and displacement are horizontal and friction is negligible, the work done is equal to the change in the kinetic energy of the object. The expression $W = \Delta E_{\mathrm{k}}$ is often called the **work-kinetic energy theorem**.

SAMPLE PROBLEM

The Hammer and Nail

You drive a nail horizontally into a wall, using a 0.448 kg hammerhead. If the hammerhead is moving horizontally at 5.5 m/s and in one blow drives the nail into the wall a distance of 3.4 cm, determine the average force acting on

(a) the hammerhead

(b) the nail

Conceptualize the Problem

- The hammer possesses *kinetic energy*.
- The *backward force* exerted by the nail on the hammer *removes* all of the *kinetic energy*.
- The *magnitude of the force* exerted by the hammer on the nail equals the *magnitude of the force* exerted by the nail on the hammer, according to Newton's third law of motion.

Identify the Goal

(a) The force acting on the hammer, $\vec{F}_h$, by the nail

(b) The force applied to the nail, $\vec{F}_n$, by the hammer

Identify the Variables and Constants

Known	Implied	Unknown
$m = 0.448\ \text{kg}$	$\vec{v}_f = 0\ \frac{\text{m}}{\text{s}}$	$\vec{F}_h$
$\vec{v}_i = 5.5\ \frac{\text{m}}{\text{s}}[\text{forward}]$		$\vec{F}_n$
$\Delta\vec{d} = 0.034\ \text{m}[\text{forward}]$		

PROBLEM TIP

When solving problems involving work and energy, be sure to express all quantities in SI units. For example, a speed of 80 km/h must be converted into 22.2 m/s, and a mass of 25 g must be expressed as 0.025 kg.

Develop a Strategy

With only horizontal motion, work done equals the change in kinetic energy.

$$|\vec{F}_h||\Delta\vec{d}| = \Delta E_k$$

$$|\vec{F}_h||\Delta\vec{d}| = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

$$|\vec{F}_h| = \frac{\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2}{\Delta d}$$

$$|\vec{F}_h| = \frac{\tfrac{1}{2}(0.448\ \text{kg})\left(0\ \tfrac{\text{m}}{\text{s}}\right)^2 - \tfrac{1}{2}(0.448\ \text{kg})\left(5.5\ \tfrac{\text{m}}{\text{s}}\right)^2}{0.034\ \text{m}}$$

$$\vec{F}_h = -199.2941\ \text{N}$$

$$\vec{F}_h \cong -2.0 \times 10^2\ \text{N}$$

(a) The average force exerted on the hammer by the nail was 2.0×10^2 N[backward].

Apply Newton's third law to the forces between the hammer and the nail.

$$\vec{F}_n = -\vec{F}_h$$

$$\vec{F}_n \cong -(-2.0 \times 10^2\ \text{N})$$

$$\vec{F}_n \cong 2.0 \times 10^2\ \text{N}$$

(b) The force exerted on the nail by the hammer was 2.0×10^2 N[forward].

continued ▶

continued from previous page

Validate the Solution

Since kinetic energy must be transferred out of the hammerhead, the force on the hammer must be in the opposite direction to its motion and so the force must be negative.

PRACTICE PROBLEMS

4. A car with a mass of 2.00×10^3 kg is travelling at 22.2 m/s (80 km/h) when the driver applies the brakes. If the force of static friction between the tires and the road is 8.00×10^3 N[backward], determine the stopping distance of the car. Use the concepts of work and energy in solving this problem.

5. A 1.00×10^2 g arrow is fired horizontally from a bow. If the average applied force on the arrow is 150.0 N and it acts over a displacement of 40.0 cm, with what speed will the arrow leave the bow? Use the concepts of work and energy in solving this problem.

6. A 12.0 kg sled is sliding at 8.0 m/s over ice when it encounters a patch of snow. If it comes to rest in 1.5 m, determine the magnitude of the average force acting on the sled.

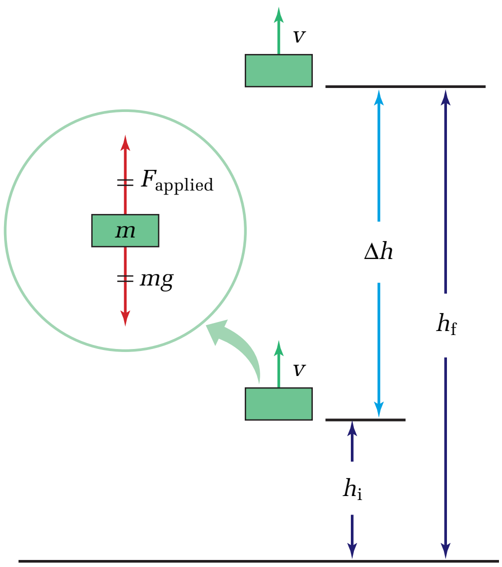


Figure 5.4 The upward force that you apply to the mass is equal in magnitude and opposite in direction to the force of gravity. Therefore, the velocity of the mass is constant and only its height changes.

Work and Gravitational Potential Energy

Consider, now, a contrasting situation — the motion of the object on which work is being done and the force that does the work are vertical and there is no change in the object's velocity. For example, imagine that you are lifting a mass at constant speed, so there is no change in its kinetic energy. The only energy that the mass will gain will be due to its position in the gravitational field — gravitational potential energy (E_g). The relationship between the quantities is shown in Figure 5.4.

Because neither the speed nor the direction of the mass is changing, it is not accelerating. If the acceleration of the mass is zero, then the net force must be zero and therefore the magnitude of upward applied force must equal the magnitude of the downward force of gravity.

$$F_{applied} = F$$
$$F_{applied} = mg$$

Examine the following steps to derive the relationship between work done by a vertical force and gravitational potential energy.

- Write the equation for work. From Figure 5.4, you can see that the angle between the force vector and the displacement vector is zero. Use this information to simplify the equation.

$$W = F\Delta d \cos\theta$$
$$W = F\Delta d \cos 0^\circ$$
$$\cos 0^\circ = 1$$
$$W = F\Delta d$$

- To find the amount of work done by the applied force, substitute the applied force into the equation.

$$W = F\Delta d$$
$$W = F_{\text{applied}}\Delta d$$
$$F_{\text{applied}} = mg$$
$$W = mg\Delta d$$

- From Figure 5.4, you can see that the displacement is the difference of the initial and final heights. Substitute this value into the equation for work.

$$\Delta d = h_{\text{f}} - h_{\text{i}}$$
$$W = mg(h_{\text{f}} - h_{\text{i}})$$

- Expand the equation.

$$W = mgh_{\text{f}} - mgh_{\text{i}}$$

- Recognize the expression *mgh* as gravitational potential energy and substitute E_{g} into the equation for work.

$$W = E_{\text{gf}} - E_{\text{gi}}$$
$$W = \Delta E_{\text{g}}$$

When velocity does not change but the object's position changes in height, the work done on an object is equal to the change in the gravitational potential energy of the object.

SAMPLE PROBLEM

A Rescue at Sea

A gas-powered winch on a rescue helicopter does 4.20×10^3 J of work while lifting a 50.0 kg swimmer at a constant speed up from the ocean. Through what height was the swimmer lifted?

Conceptualize the Problem

- The *speed* was *constant*, so there was *no change* in *kinetic energy*.
- Assume that the *work done* by the winch equals the gain in *gravitational potential energy* of the swimmer.

Identify the Goal

The height, Δh, through which the swimmer was lifted

continued ►

continued from previous page

Identify the Variables and Constants

Known	Implied	Unknown
$W = 4.20 \times 10^3$ J	$g = 9.81\ \frac{\text{m}}{\text{s}^2}$	Δh
$m = 50.0$ kg		

Develop a Strategy

With no change in kinetic energy, work done equals the change in gravitational potential energy. Substitute and solve.

$$W = \Delta E_g$$
$$W = mg\Delta h$$
$$\Delta h = \frac{W}{mg}$$
$$\Delta h = \frac{4.20 \times 10^3 \text{ J}}{(50.0 \text{ kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$
$$\Delta h = 8.562\ 69 \text{ m}$$
$$\Delta h \cong 8.56\ m$$

The height through which the swimmer was lifted was 8.56 m.

Validate the Solution

$1 \text{ J} = \dfrac{1 \text{ kg}\cdot\text{m}^2}{\text{s}^2}$, so the answer has units of $\dfrac{\frac{1\ \cancel{\text{kg}}\cdot\text{m}^{\cancel{2}}}{\cancel{\text{s}^2}}}{\cancel{\text{kg}}\cdot\frac{\cancel{\text{m}}}{\cancel{\text{s}^2}}} = \text{m}$.

PRACTICE PROBLEMS

7. A motorized crane did 40.4 kJ of work when slowly lifting a pile driver to a height of 8.00 m. What was the mass of the pile driver?

8. A 4.00×10^2 kg elevator car rose at a constant speed past several floors. If the motor did 58.8 kJ of work, through what height did the car rise?

9. A battery-powered scoop used by a Mars lander lifted a 54 g rock through a height of 24 cm. If $g_{\text{Mars}} = 3.8$ m/s^2, how much work was done by the scoop?

The Work-Energy Theorem and Conservation of Energy

Very few processes are as limited as the two situations that you have just considered — changes in kinetic energy only or in potential energy only. Real processes usually involve more than one form of energy. However, you can combine the two cases that you just considered. For example, if an applied force does work on an object so that both its kinetic energy and its various forms of potential energy change, then the work done by that force equals the total change in both the kinetic energy and the potential energies: $W = \Delta E_k + \Delta E_p$. The relationship shown in this equation is known as the **work-energy theorem**.

Picture an object or a system of objects on which no work is being done by some outside agency. In other words, no energy is being added to the system and no energy is being removed from the system. This is called an "isolated system." A swinging pendulum could be such a system until someone comes along and gives it a shove. A roller coaster in which the car is running freely up and down slopes and around curves is isolated if wind effects and friction are ignored. The flight of an arrow away from its position in a stretched bow can be treated as an isolated system if air friction effects and wind are again ignored.

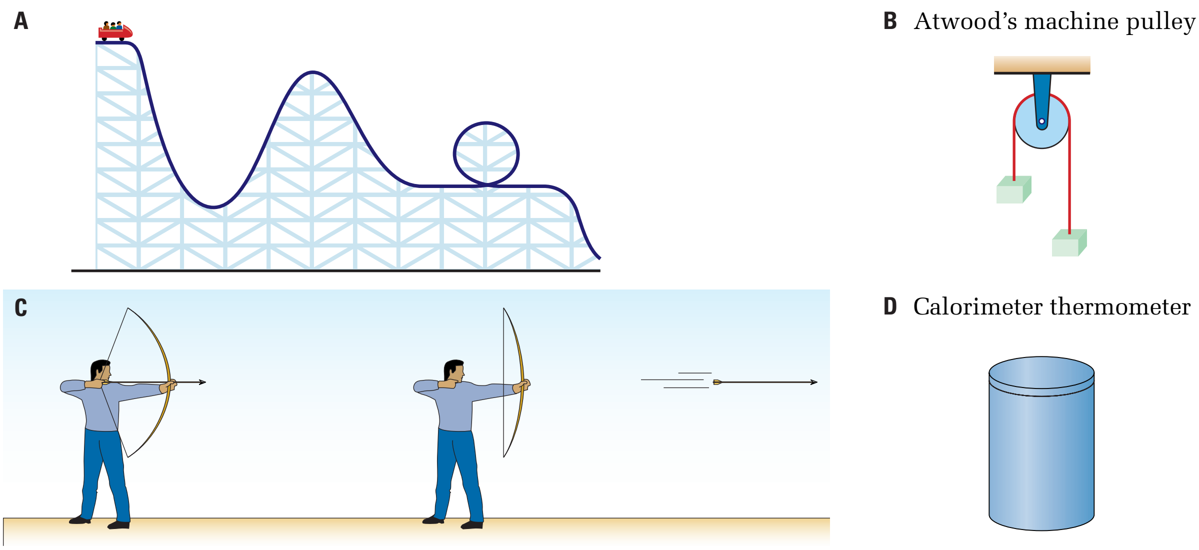


Figure 5.5 Our universe is possibly the only truly isolated system. However, in many applications, such as the ones shown here, you will work on the assumption that no energy enters from the outside and none is lost to the outside.

If a system is isolated in that no outside work is done on it, you can use the work-energy theorem to derive another important relationship, as shown in the following steps.

- Write the work-energy theorem. $W = \Delta E_k + \Delta E_p$
- If the system is isolated, $W = 0$. Substitute zero into the equation above. $0 = \Delta E_k + \Delta E_p$; $\Delta E_k + \Delta E_p = 0$
- Expand the expression and use primes to represent the energies after the process is complete. $(E_k' - E_k) + (E_p' - E_p) = 0$
- Rearrange the equation so that the initial energies are on the left-hand side of the equals sign and all of the final energies are on the right-hand side. $E_k + E_p = E_k' + E_p'$

PHYSICS FILE

The work-energy theorem links two apparently different types of quantities. On the left-hand side is a concept that is extremely concrete in that it deals with readily measured quantities of force and distance. On the right-hand side is the extremely abstract concept of energy. In fact, the right-hand side deals *only* with energy changes and never in absolute amounts of energy.

ELECTRONIC LEARNING PARTNER

Use the interactive pendulum simulation in your Electronic Learning Partner to enhance your understanding of energy transformations.

The last statement in the derivation is known as the law of **conservation of mechanical energy**. The equation $\Delta E_k + \Delta E_p = 0$ says that the change of total mechanical energy in an isolated system is zero. This does not mean, however, that no processes are occurring within the system. This last statement implies that kinetic energy of an object in the system can be transformed into potential energy, or the reverse can happen. In addition, one object in the system might transfer energy to another object in the system. Many processes can occur in an isolated system.

THE LAW OF CONSERVATION OF MECHANICAL ENERGY

The law of conservation of mechanical energy states that the sum of the kinetic and potential energies before a process occurs in an isolated system is equal to the sum of the kinetic and potential energies of the system after the process is complete.

$$E_k + E_p = E'_k + E'_p$$

Quantity	Symbol	SI unit
kinetic energy before the process occurred	E_k	J (joules)
potential energy before the process occurred	E_p	J (joules)
kinetic energy after the process was completed	E'_k	J (joules)
potential energy after the process was completed	E'_p	J (joules)

PROBEWARE

If your school has probeware equipment, visit ***www.mcgrawhill.ca/links/physics12*** and follow the links for an in-depth activity on energy and momentum in collisions.

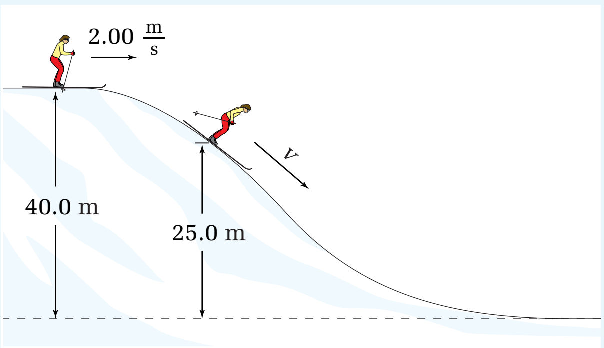
SAMPLE PROBLEM

Conservation of Energy on the Ski Slopes

A skier is gliding along with a speed of 2.00 m/s at the top of a ski hill, 40.0 m high, as shown in the diagram. The skier then begins to slide down the icy (frictionless) hill.

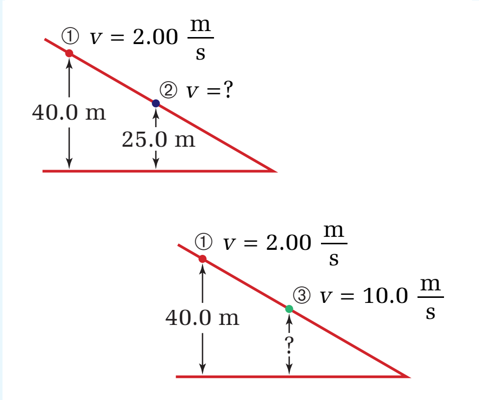
(a) What will be the skier's speed at a height of 25.0 m?

(b) At what height will the skier have a speed of 10.0 m/s?



Conceptualize the Problem

- Sketch the two parts of the problem separately. Label the initial conditions (top of the hill) "1." Label the position when h = 25 m as "2." Label the position at which the skier is travelling at 10.0 m/s as "3."
- Use subscripts 1 and 2 to indicate the initial and final conditions in part (a) and use subscripts 1 and 3 to indicate the initial and final conditions in part (b).
- Define the *system* as the *skier and the slope*.
- Assume that the system of skier and slope is *isolated*.
- The law of *conservation of energy* can be applied.



Identify the Goal

(a) the speed, v_2, at a height of 25.0 m

(b) the height, h_3, at which the skier's speed will be 10.0 m/s

Identify the Variables and Constants

Known	Implied	Unknown
$v_1 = 2.00\ \frac{m}{s}$	$g = 9.81\ \frac{m}{s^2}$	v_2
$h_1 = 40.0$ m		h_3
$v_3 = 10.0\ \frac{m}{s}$		
$h_2 = 25.0$ m		

Develop a Strategy

State the law of conservation of mechanical energy.

$$E'_k + E'_p = E_k + E_p$$

Expand by replacing E with the expression that defined the type of energy.

$$\frac{1}{2}mv_2^2 + mgh_2 = \frac{1}{2}mv_1^2 + mgh_1$$

Divide through by m.

$$\frac{1}{2}v_2^2 = \frac{1}{2}v_1^2 + gh_1 - gh_2$$

Simplify and rearrange the equation to solve for v_2.

$$v_2^2 = v_1^2 + 2g(h_1 - h_2)$$

$$v_2 = \sqrt{v_1^2 + 2g(h_1 - h_2)}$$

Substitute and solve.

$$v_2 = \sqrt{\left(2.00\ \frac{m}{s}\right)^2 + 2\left(9.81\ \frac{m}{s^2}\right)(40.0\ m - 25.0\ m)}$$

$$v_2 = \sqrt{298.3\ \frac{m^2}{s^2}}$$

Speed does not involve direction, so choose the positive root since speed can never be negative.

$$v_2 = \pm 17.271\ \frac{m}{s}$$

$$v_2 \cong 17.3\ \frac{m}{s}$$

(a) The speed is 17.3 m/s at a height of 25.0 m.

continued ▶

continued from previous page

Write the expanded version of the conservation of mechanical energy. Rearrange and solve for height.

$$\frac{1}{2}mv_3^2 + mgh_3 = \frac{1}{2}mv_1^2 + mgh_1$$

$$gh_3 = \frac{1}{2}v_1^2 + gh_1 - \frac{1}{2}v_3^2$$

$$h_3 = \frac{\frac{1}{2}v_1^2 + gh_1 - \frac{1}{2}v_3^2}{g}$$

Substitute numerical values and solve.

$$h_3 = \frac{\frac{1}{2}\left(2.00\ \frac{\text{m}}{\text{s}}\right)^2 + \left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(40.0\ \text{m}) - \frac{1}{2}\left(10.0\ \frac{\text{m}}{\text{s}}\right)^2}{9.81\ \frac{\text{m}}{\text{s}^2}}$$

$$h_3 = 35.107\ \text{m}$$

$$h_3 \cong 35.1\ \text{m}$$

(b) The height must be 35.1 m when the speed is 10.0 m/s.

Validate the Solution

(a) The units for the right-hand side of the equation for v_2 are

$$\left[\left(\frac{\text{m}}{\text{s}}\right)^2 + \left(\frac{\text{m}}{\text{s}^2}\right)\text{m}\right]^{\frac{1}{2}} = \frac{\text{m}}{\text{s}}.$$

These are the correct units for the speed. In addition, since the skier is going downhill, the final speed must be larger than the initial speed of 2.00 m/s.

(b) The units on the right-hand side of the equation for h_3 are

$$\frac{\left(\frac{\text{m}}{\text{s}}\right)^2 + \left(\frac{\text{m}}{\text{s}^2}\right)(\text{m}) - \left(\frac{\text{m}}{\text{s}}\right)^2}{\frac{\text{m}}{\text{s}^2}} = \text{m}.$$

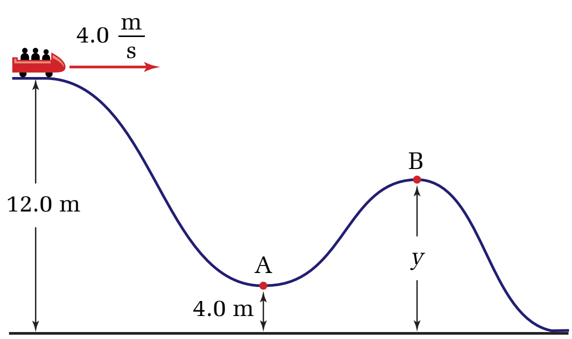
Since the speed in part (b) is less than the speed in part (a), the skier should be higher on the hill than in part (a).

PROBLEM TIP

Students are often tempted to apply the equations for linear motion to the solution of these problems. However, the paths are not always linear, so the equations might not apply. One of the great advantages of using conservation of energy is that you generally do not need to know the exact path between two points or vector directions. You need to know the conditions at only those two points.

PRACTICE PROBLEMS

Use the following diagram for practice problems 10, 11, and 12.



10. A car on a roller coaster is moving along a level section 12.0 m high at 4.0 m/s when it begins to roll down a slope, as shown in the diagram. Determine the speed of the car at point A.

11. What is the height of point B in the roller coaster track if the speed of the car at that point is 10.0 m/s?

12. What is the height of y if the speed of the roller coaster at B is 12.5 m/s?